JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-19, No.-10, October (2024) pp 67 - 86 ISSN (Print) 0973-8975

TRANSMISSION AND CONTROL DYNAMICS OF ROTAVIRUS DIARRHEA MODEL WITH DOUBLE DOSE VACCINATION

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https://doi.org/10.26782/jmcms.2024.10.00005

(Received: July 31, 2024; Revised: September 26, 2024; Accepted: October 06, 2024)

Abstract

This study introduces a six-compartmental mathematical model (S, V_1, V_2, E, I, R) to examine the impact of administering a double dose vaccine on the dynamic spread of diarrhea within a community. The mathematical analysis shows the existence of equilibrium points for both disease-free and endemic states in the model. The basic reproduction number R_o was determined using the Next Generation Matrix. Analysis has shown that the basic reproduction number $R_o < 1$ which indicates the disease-free equilibrium point is locally asymptotically stable. Also, using a suitable Lyapunov functional for the model system expressed in state variables and parameters defining the dynamic characteristics of spread and control strategies of the rotavirus diarrhea to obtain the global stability of disease-free equilibrium point over time. A numerical simulation was carried out by Wolfram Mathematica to show the effect of a second-dose vaccine. The inclusion of a double-dose vaccine has been found to have a significant effect on completely eliminating the outbreak of diarrhea. This is evidenced by the local and global stability results, which indicate that effective measures have been taken to prevent the reintroduction or transmission of the disease, and if there may be a risk of outbreaks or reemergence of the disease, very little continuous monitoring and intervention strategies are required to maintain control as this should be taken seriously by medical practitioners or policy health makers.

Keywords: Stability, Basic Reproduction Number, Vaccination; Diarrhea Model, Lyapunov function,

I. Introduction

Diarrhea is the second most common cause of death in children under five years old, claiming the lives of numerous children worldwide. It causes more deaths among young children than any other childhood infectious diseases, with over 1.5

million deaths attributed to diarrhea annually. This makes it a more significant health threat in terms of mortality than infectious diseases. It is a health issue characterized by frequent loose and watery bowel movements, often accompanied by abdominal bloating and pressure. The primary cause of diarrhea is typically an infection within the digestive system, which can be attributed to various microorganisms like viruses, bacteria, or parasites. Rotavirus is a highly contagious virus that frequently leads to diarrhea in infants and young children. It is disseminated by contact with an infected person's stool with symptoms like severe watery diarrhea, vomiting, fever, and abdominal pain. Infected children may require hospitalization due to dehydration. The CDC recommends rotavirus vaccination for infants to prevent the disease. Study shows that most of these vaccines do not give complete protection because their effectiveness wanes over time. Therefore, this study aims to investigate the potential benefits of administering second doses of rotavirus vaccines to individuals to provide additional protection against the disease.

The historical usage of differential equations in modeling biological, ecological, and medical systems can be traced back to notable figures such as Verhulst, Malthus, Lotka, and Volterra, see [IX]. Over the years ordinary differential equations have been useful for modeling natural phenomena. Specifically, systems of nonlinear differential equations have shown great effectiveness in capturing population dynamics, the spread of infectious diseases, inter-species interactions, and various other biological processes, see ([VI], [VIII], [XI], [XII] and [XIII]). [VII] conducted research on the assessment of controlling bovine viral diarrhea virus using a mathematical model that captures the dynamics of infection. According to the model's predictions, approximately 1.2 percent of animals would have persistent infections, aligning with field estimates. Moreover, the model displayed limited sensitivity to modifications in its structure or parameter values. This study provided valuable insights into Bovine Viral Diarrhea (BVD) control measures, highlighting the significant role played by persistently infected (PI) animals in perpetuating BVD as an ongoing issue within a herd. [III] studied an epidemiological model of diarrhea diseases and its potential for prevention and control. The model successfully replicated the observed patterns of infantile diarrhea diseases, which are primarily linked to enterotoxigenic Escherichia coli or rotavirus. Through the proposed mathematical model, they were able to predict a plausible serological profile of an enteric infection while [V] studied the impact of saturation treatment on the dynamic spread of diarrhea in the community using a mathematical (SITR) model. [I] investigated the impact of a vaccine on cases of diarrhea. They calculated the basic reproduction number (R_o), and found that when $R_o > 1$, the disease became endemic, indicating that it persisted within the population at a steady rate as each infected person transmitted the disease to one susceptible individual. Recently, [XIV] examined how vaccines and treatment impact the spread of diarrhea in a community. They demonstrated that the model has a disease-free equilibrium point that is stable both locally and globally over time. The findings illustrated that vaccines and treatment significantly decrease the occurrence of diarrhea infections. However, they noted that while vaccination alone is not enough to reduce the basic reproduction number, it does manage the disease.

Despite multiple efforts, eliminating diarrhea has proven to be a difficult undertaking

due to ongoing infections even in the presence of a vaccine, see [II], [IV] and other literature cited therein. To investigate the impact of a double-dose vaccine on the dynamic spread of diarrhea in a community, we developed a deterministic epidemic model (S, V_1, V_2, E, I, R). The findings indicate that if the basic reproduction number is below one, regardless of the initial number of infected individuals in the population, diarrhea can be controlled through the use of a double-dose vaccine rapidly and over time.

II. Model Equation

In this context, we examine six categories of people, namely those who are susceptible (S), have received one dose of vaccine (V_1), have received two doses of vaccine (V_2), are exposed to the disease (E), are infected (I), and have recovered from the disease (R). This model is suitable for diseases that involve a significant period after infection during which an exposed person is not yet capable of spreading the infection. The S, V_1, V_2, E, I, R model comprises a collection of six differential equations.

$$\begin{aligned}
\frac{dS}{dt} &= (1-\rho)\pi - \beta SI - \mu S + \theta R \\
\frac{dV_1}{dt} &= \rho\pi - \epsilon\beta V_1 I - (\mu+\alpha)V_1 \\
\frac{dV_2}{dt} &= \alpha V_1 - (\mu+\phi)V_2 \\
\frac{dE}{dt} &= \beta SI + \epsilon\beta V_1 I - (\mu+\sigma)E \\
\frac{dI}{dt} &= \sigma E - (\mu+\delta+\gamma+\tau)I \\
\frac{dR}{dt} &= \phi V_2 + (\gamma+\tau)I - (\mu+\theta)R
\end{aligned} \tag{1}$$

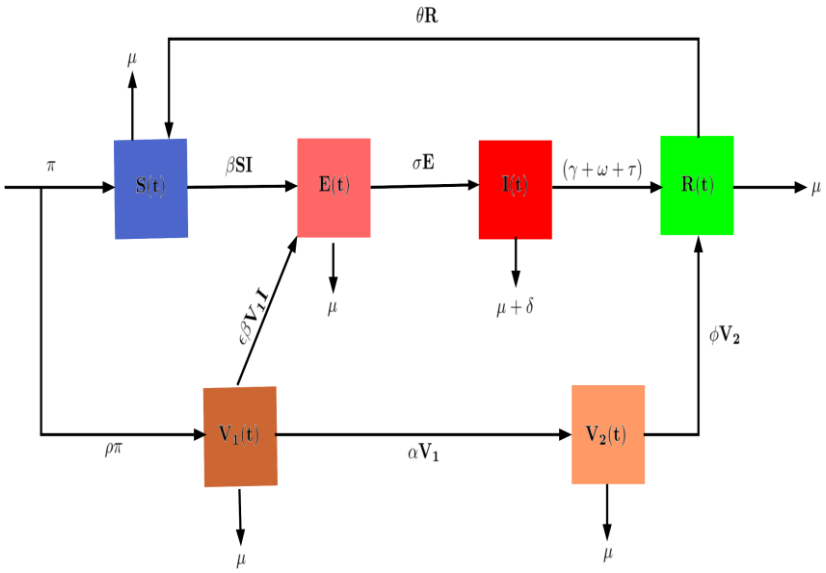


Fig 1. Schematic Diagram for the Diarrhea Model

Table 1: Description of parameters of the model

Parameter	Description
π	Recruitment rate
β	Contact rate
ρ	Vaccination rate
μ	Natural death rate
θ	The rate at which recovered individuals revert to susceptible class
ϵ	Reduction in infectivity due to first dose vaccine
α	Rate of second dose vaccination
ϕ	Rate of acquiring immunity from second dose vaccine
τ	Treatment rate of exposed persons
σ	The infection rate of the exposed person
δ	Induced disease death rate
γ	Natural recovery rate

III. Disease-Free Equilibrium

At disease-free equilibrium, there's no infection i.e. $I = E = 0$. At the equilibrium point, the model formulation is set to zero.

$$\frac{dS}{dt} = \frac{dV_1}{dt} = \frac{dV_2}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0.$$

Solving for S, V_1, V_2, E and R, give the disease-free equilibrium $\xi_o = (S^o, V_1^o, V_2^o, E^o, R^o)$ as

$$\xi_0 = \frac{(1-\rho)\pi}{\mu} + \frac{\phi\alpha\rho\pi}{\mu(\mu+\theta)(\mu+\phi)(\mu+\alpha)}, \frac{\rho\pi}{\mu+\alpha}, \frac{\alpha\rho\pi}{(\mu+\phi)(\mu+\alpha)}, 0, 0, \frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}.$$

IV. Basic Reproduction Number R_O/R_e

Here, the effective reproduction number is influenced by the inclusion of two doses of vaccines as a control measure in the model.

To compute the effective reproduction number, R_e, we employed the method of next generation matrix in which $R_e = \rho(FV^{-1})$ where

$$F = D\mathcal{F}|_\epsilon = D\begin{pmatrix}\beta SI + \epsilon\beta V_1 I \\ 0\end{pmatrix}_\epsilon = \begin{pmatrix}0 & \frac{\beta}{\mu}\left[(1-\rho)\pi + \frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)} + \frac{\epsilon\beta\rho\pi}{\mu+\alpha}\right] \\ 0 & 0\end{pmatrix}$$

$$V = D\mathcal{V}|_\epsilon = D\begin{pmatrix}(\mu+\sigma)E \\ -\sigma E + (\mu+\delta+\gamma+\tau)\end{pmatrix}_\epsilon = \begin{pmatrix}\mu+\sigma & 0 \\ -\sigma & \mu+\delta+\gamma+\tau\end{pmatrix}$$

$$V^{-1} = \frac{1}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)}\begin{pmatrix}\mu+\delta+\gamma+\tau & 0\\ \sigma & \mu+\sigma\end{pmatrix}$$
$$= \begin{pmatrix}\frac{1}{\mu+\delta} & 0\\ \frac{\sigma}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)} & \frac{1}{\mu+\delta+\gamma+\tau}\end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix}0 & \frac{\beta}{\mu}\left[(1-\rho)\pi + \frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}\right] + \frac{\epsilon\beta\rho\pi}{\mu+\alpha}\\ 0 & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\mu+\delta} & 0\\ \frac{\sigma}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)} & \frac{1}{\mu+\delta+\gamma+\tau}\end{pmatrix}$$
$$= \begin{pmatrix}\frac{\beta\sigma}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)}\Phi & \frac{\beta}{\mu+\delta+\gamma+\tau}\Phi\\ 0 & 0\end{pmatrix}$$

where

$$\Phi = \left\{\frac{1}{\mu}\left[(1-\rho)\pi + \frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}\right] + \frac{\epsilon\rho\pi}{\mu+\alpha}\right\}$$

We then solve $|FV^{-1} - \lambda I| = 0$ to find the eigenvalues λ i.e

$$\begin{vmatrix}\frac{\beta\sigma}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)}\Phi - \lambda & \frac{\beta}{\mu+\delta+\gamma+\tau}\Phi\\ 0 & -\lambda\end{vmatrix} = 0$$

$$\Rightarrow$$

$$\lambda = 0$$

or

$$\lambda = \frac{\beta\sigma\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)}\left[(1-\rho)(\mu+\alpha) + \frac{\theta\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)} + \mu\epsilon\rho\right] = R_e$$

Hence,

$$R_e = \frac{\beta\sigma\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)}\left[(1-\rho)(\mu+\alpha) + \frac{\theta\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)} + \mu\epsilon\rho\right]$$

In the absence of vaccination, the basic reproduction number (i.e if $\rho = \alpha = 0$) is

$$R_o = \frac{\beta\sigma\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)}.$$

V. Endemic Equilibrium Points

At the state of endemic equilibrium, there is an infection present within the host population, meaning that $E, I \neq 0$. To achieve an endemic equilibrium, we need to set each equation in the formulated model to zero in Eq. (1) as

$$(1-\rho)\pi - \beta SI - \mu S + \theta R = 0 \qquad (2)$$

$$\rho\pi - \epsilon\beta V_1 I - (\mu + \alpha)V_1 = 0 \quad (3)$$

$$\alpha V_1 - (\mu + \phi)V_2 = 0 \quad (4)$$

$$\beta SI + \epsilon\beta V_1 I - (\mu + \sigma)E = 0 \quad (5)$$

$$\sigma E - (\mu + \delta + \gamma + \tau)I = 0 \quad (6)$$

$$\phi V_2 + (\gamma + \tau)I - (\mu + \theta)R = 0 \quad (7)$$

From Eqs. (4) and (7);

$$V_2 = \frac{\alpha}{\mu+\phi} V_1$$

$$E = \frac{\mu+\delta+\gamma+\tau}{\sigma} I$$

substitute for V_2 and I in (7), we have

$$\frac{\alpha\phi}{\mu+\phi} V_1 + \frac{\sigma(\gamma+\tau)}{(\mu+\delta+\gamma+\tau)} E - (\mu + \theta)R = 0$$

$$\Longrightarrow$$

$$R = \frac{1}{\mu+\theta}\left[\frac{\alpha\phi V_1}{\mu+\phi} + (\delta + \tau)I\right].$$

From (3)

$$\rho\pi - (\epsilon\beta I + \mu + \alpha)V_1 = 0 \Longrightarrow$$

$$V_1 = \frac{\rho\pi}{\epsilon\beta I+\mu+\alpha}$$

From (2)

$$(1 - \rho)\pi - (\beta I + \mu)S + \theta R = 0 \Longrightarrow$$

$$S = \frac{(1-\rho)\pi+\theta R}{\beta I+\mu} \quad (8)$$

Also from (5)

$$S = \frac{(\mu+\sigma)E-\epsilon\beta V_1 I}{\beta I} \quad (9)$$

From (8) and (9), we have

$$\frac{(1-\rho)\pi+\theta R}{\beta I+\mu} = \frac{(\mu+\sigma)E-\epsilon\beta V_1 I}{\beta I}$$

substituting for V_1, E and R yields

$$(1-\rho)\pi\beta I+\frac{\theta\beta I}{\mu+\phi}\left[\frac{\alpha\phi V_1}{\mu+\phi}+(\gamma+\tau)I\right]=\frac{(\mu+\sigma)\beta I(\mu+\delta+\gamma+\tau)}{\sigma}I$$
$$+\frac{\mu+(\mu+\sigma)(\mu+\delta+\gamma+\tau)}{\sigma}I-\frac{\epsilon\rho\pi}{\epsilon\beta I+\mu+\alpha}(\beta I)^2-\frac{\mu\epsilon\beta\rho\pi I}{\epsilon\beta I+\mu+\alpha}.$$

$$(1-\rho)\pi\beta I+\frac{\theta\beta I\cdot\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\epsilon\beta I+\mu+\alpha)}+\frac{\theta\beta(\gamma+\tau)}{\mu+\theta}I=\frac{(\mu+\sigma)(\mu+\delta+\gamma+\tau)\beta I}{\sigma}I$$
$$+\frac{\mu+(\mu+\sigma)(\mu+\delta+\gamma+\tau)}{\sigma}I-\frac{\epsilon\rho\pi}{\epsilon\beta I+\mu+\alpha}(\beta I)^2-\frac{\mu\epsilon\beta\rho\pi I}{\epsilon\beta I+\mu+\alpha},$$

multiplying through by $\epsilon\beta I+\mu+\alpha$ to obtain

$$(1-\rho)\pi\epsilon\beta^2I^2+(1-\rho)(\mu+\alpha)\pi\beta I+\frac{\theta\beta\alpha\phi\rho\pi I}{(\mu+\theta)(\mu+\phi)}I+\frac{\theta\beta(\gamma+\tau)E\beta I^2}{\mu+\theta}$$
$$+\frac{\theta\beta(\gamma+\tau)(\mu+\alpha)I}{\mu+\theta}=\frac{\beta(\mu+\sigma)(\mu+\delta+\gamma+\tau)\epsilon\beta I^2}{\sigma}$$
$$+\frac{\beta(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)I^2}{\sigma}+\frac{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)\epsilon\beta I^2}{\sigma}$$
$$+\frac{\mu(\mu+\sigma)(\mu+\delta\gamma+\tau)(\mu+\alpha)I}{\sigma}-\epsilon\rho\pi\beta^2I^2-\mu\epsilon\rho\pi\beta I.$$

$$\frac{(\mu+\sigma)(\mu+\delta+\gamma+\tau)\epsilon\beta^2I^3}{\sigma}$$
$$+\left[\frac{(\mu+\sigma)(\mu+\delta+\gamma+\tau)\beta}{\sigma}(\mu+\alpha+\epsilon\mu)-(1-\rho)\pi\epsilon\beta^2-(\gamma+\tau)\epsilon\theta\beta^2-\epsilon\rho\pi\beta^2\right]I^2$$
$$+\left[\frac{(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)\mu}{\sigma}\right.$$
$$\left.-(1-\rho)(\mu+\alpha)\pi\beta-\frac{\theta\beta}{\mu+\theta}\left(\frac{\alpha\phi\rho\pi}{\mu+\phi}+(\gamma+\tau)(\mu+\alpha)\right)-\mu\epsilon\rho\pi\beta\right]I=0$$

$\Longrightarrow I=0$ or

$$\epsilon\beta^2(\mu+\sigma)(\mu+\delta+\gamma+\tau)I^2$$
$$+\left[\beta(\mu+\delta)(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha+\epsilon\mu)-\sigma\epsilon\beta^2\left(\epsilon\rho\pi+\theta(\gamma+\tau)\right)\right]I$$
$$+\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)$$
$$\left[1-\frac{\beta\sigma\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)}\left[(1-\rho)(\mu+\alpha)+\mu\epsilon\rho+\frac{\theta\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)}\right]\right.$$
$$\left.-\frac{\theta\beta\sigma(\gamma+\tau)(\mu+\alpha)}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)(\mu+\theta)}\right]=0.$$

It follows that

$$\begin{aligned}&\epsilon\beta^2(\mu+\sigma)(\mu+\delta+\gamma+\tau)I^2\\&+\beta(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha+\epsilon\mu)\left[1-\frac{\epsilon\beta^2\sigma\left(\rho\pi+\theta(\gamma+\tau)\right)}{\beta(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha+\epsilon\mu)}\right]I\\&-\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)\left[\frac{\theta\beta\sigma(\gamma+\tau)(\mu+\alpha)}{\mu(\mu+\delta)(\mu+\delta+\gamma+\tau)(\mu+\alpha)(\mu+\theta)}+R_e-1\right]=0,\end{aligned}\tag{10}$$

where R_e is the effective reproduction number obtained in Section 3.
From (10), if

$$\frac{\epsilon\beta^2\sigma\left(\rho\pi+\theta(\gamma+\tau)\right)}{\beta(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha+\epsilon\mu)}<1$$

and if

$$\frac{\theta\beta\sigma(\gamma+\tau)(\mu+\alpha)}{\mu(\mu+\delta)(\mu+\delta+\gamma+\tau)(\mu+\alpha)(\mu+\theta)}+R_e>1$$

then (10) has only one sign change and so, only one positive root. Hence the model has a unique equilibrium point, defined by

$$\mathcal{E}_1=(S^*,V_1^*,V_2^*,E^*,I^*,R^*),\quad \text{where}$$

$$\begin{aligned}S^* &= \frac{1}{\beta I+\mu}\left\{(1-\rho)\pi+\frac{\theta}{\mu+\theta}\left[\frac{\alpha\phi\rho\pi}{(\mu+\phi)(\epsilon\beta I+\mu+\alpha)}+(\gamma+\tau)I\right]\right\}\\ V_1^* &= \frac{\rho\pi}{\epsilon\beta I+\mu+\alpha}\\ V_2^* &= \frac{\alpha\rho\pi}{(\mu+\phi)(\epsilon\beta I+\mu+\alpha)}\\ E^* &= \frac{\mu+\delta\gamma+\tau}{\sigma}I\\ R^* &= \frac{1}{\mu+\theta}\left[\frac{\alpha\phi\rho\pi}{(\mu+\phi)(\epsilon\beta I+\mu+\alpha)}+(\gamma+\tau)I\right]\end{aligned}$$

and I^* is the positive root of the quadratic equation defined in (10).

VI. Local Stability of the Disease-Free Equilibrium (DFE)

Theorem 1. *The determined disease-free equilibrium*

$$\xi_0=\frac{(1-\rho)\pi}{\mu}+\frac{\phi\alpha\rho\pi}{\mu(\mu+\theta)(\mu+\phi)(\mu+\alpha)},\frac{\rho\pi}{\mu+\alpha},\frac{\alpha\rho\pi}{(\mu+\phi)(\mu+\alpha)},$$
$$0,0,\frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}.$$

applies for non-negative values of its parameters with $R_o\leq 1$ and $R_o>1$ being locally asymptotically stable and unstable respectively.

Proof: From equation Eq. (1), it follows that

$$(1-\rho)\pi - \beta SI - \mu S + \theta R = 0$$

$$\rho\pi - \epsilon\beta V_1 I - (\mu+\alpha)V_1 = 0$$

$$\alpha V_1 - (\mu+\phi)V_2 = 0$$

$$\beta SI + \epsilon\beta V_1 I - (\mu+\sigma)E = 0$$

$$\sigma E - (\mu+\delta+\gamma+\tau)I = 0$$

$$\phi V_2 + (\gamma+\tau)I - (\mu+\theta)R = 0$$

We obtain the Jacobian of the above system of equations concerning S, V_1, V_2, E, I, R as follows,

$$J_{DFE} =$$

$$\begin{pmatrix} -\beta I_o - \mu & 0 & 0 & 0 & -\beta S_o & \theta \\ 0 & -\epsilon\beta I - (\mu+\alpha) & 0 & 0 & 0 - \epsilon\beta V_1 & 0 \\ 0 & \alpha & -(\mu+\phi) & 0 & 0 & 0 \\ \beta I_o & \epsilon\beta I_o & 0 & -(\mu+\sigma) & \beta S_o + \epsilon\beta V_1 & 0 \\ 0 & 0 & 0 & \sigma & -(\mu+\delta+\gamma+\tau) & 0 \\ 0 & 0 & \phi & 0 & (\gamma+\tau 0) & -(\mu+\theta) \end{pmatrix}$$

Solving $|J_{DFE} - I\lambda|$ at the Disease Free Equilibrium (DFE) point gives,

$$\begin{pmatrix} -(\mu+\lambda) & 0 & 0 & 0 & -\frac{\beta}{\mu}\mathcal{W} & 0 \\ 0 & -(\mu+\alpha+\lambda) & 0 & 0 & -\frac{\epsilon\beta\rho\pi}{\mu+\alpha} & 0 \\ 0 & \alpha & -(\mu+\phi+\lambda) & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\mu+\theta+\lambda) & -\frac{\beta}{\mu}\mathcal{Q}_1 & 0 \\ 0 & 0 & 0 & \sigma & -\mathcal{T} & 0 \\ 0 & 0 & \phi & 0 & (\gamma+\tau) & -(\mu+\theta+\lambda) \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{T} &= (\mu+\delta+\gamma+\tau+\lambda) \\ \mathcal{W} &= \left[(1-\rho)\pi + \frac{\theta\phi\alpha\epsilon\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}\right] \\ \mathcal{Q}_1 &= \left\{(1-\rho)\pi + \frac{\theta\phi\alpha\epsilon\pi}{(\mu+\theta)(\mu+\theta)(\mu+\alpha)} + \frac{\epsilon\rho\pi}{\mu+\alpha}\right\}. \end{aligned}$$

And evaluating the determinant we get,

$$(\mu+\lambda)(\mu+\alpha+\lambda)\left\{-(\mu+\phi+\lambda)(\mu+\sigma+\lambda)(\mu+\delta+\gamma+\tau+\lambda)-\frac{\sigma\beta\pi}{\mu}\right.$$
$$\left.\times\left\{(1-\rho)+\frac{\theta\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}\right\}+\frac{\epsilon\rho\pi}{\mu+\alpha}\right\}-(\mu+\theta+\lambda)=0,$$

$$\lambda_1=-\mu \qquad \lambda_3=-(\mu+\phi)$$
$$\lambda_2=-(\mu+\alpha) \qquad \lambda_4=-(\mu+\theta)$$

and

$$(\mu+\sigma+\lambda)(\mu+\delta+\gamma+\tau+\lambda)-\frac{\sigma\beta\pi}{\mu}\left\{(1-\rho)+\frac{\theta\phi\alpha\rho}{(\mu_\theta)(\mu+\phi)(\mu+\alpha)}+\frac{\mu\epsilon\rho}{\mu+\alpha}\right\}$$
$$=0.$$

From the immediate above equation,

$$\lambda^2+\lambda(2\mu+\sigma+\delta+\gamma+\tau)+\mu(\mu+\delta+\gamma+\tau+\sigma)+\sigma(\delta+\gamma+\tau)$$
$$-\frac{\sigma\beta\pi}{\mu}\left\{(1-\rho)+\frac{\theta\phi\alpha\rho}{(\mu_\theta)(\mu+\phi)(\mu+\alpha)}+\frac{\mu\epsilon\rho}{\mu+\alpha}\right\}=0.$$

We solve for the roots as

$$\lambda_{5,6}=\frac{-2(\mu+\sigma+\delta+\gamma+\tau)\pm\sqrt{(2\mu+\sigma+\delta+\gamma+\tau)^2-4\mathcal{Q}_2}}{2}$$

$$\lambda_5=-2(\mu+\sigma+\delta+\gamma+\tau)-\sqrt{(2\mu+\sigma+\delta+\gamma+\tau)^2-4\mathcal{Q}_2}$$

where

$$\mathcal{Q}_2=\mu(\mu+\delta+\gamma+\tau+\sigma)+\sigma(\delta+\gamma+\tau)$$
$$-\frac{\sigma\beta\pi}{\mu}\left\{(1-\rho)+\frac{\theta\phi\alpha\rho}{(\mu_\theta)(\mu+\phi)(\mu+\alpha)}+\frac{\mu\epsilon\rho}{\mu+\alpha}\right\}$$

and

$$\lambda_6=-2(2\mu+\sigma+\delta+\gamma+\tau)+$$
$$\sqrt{(2\mu+\sigma+\delta+\gamma+\tau)^2-4\left\{\mu(\mu+\delta+\gamma+\tau+\sigma)+\sigma(\delta+\gamma+\tau)-\frac{\sigma\beta\pi}{\mu}\right\}\mathcal{Q}_3}$$

where

$$\mathcal{Q}_3=\left\{(1-\rho)+\frac{\theta\alpha\rho}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}+\frac{\mu\epsilon\rho}{\mu+\alpha}\right\}.$$

Squaring both sides

$$-4\left\{\mu(\mu+\delta+\gamma+\tau+\sigma)+\sigma(\delta+\gamma+\tau)-\frac{\sigma\beta\pi}{\mu}\right.$$
$$\left.\times\left((1-\rho)+\frac{\theta\alpha\rho}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}+\frac{\mu\epsilon\rho}{\mu+\alpha}\right)\right\}=0.$$

$$\frac{\sigma\beta\pi}{\mu(\mu+\alpha)}\left((1-\rho)(\mu+\alpha)+\frac{\theta\alpha\rho}{(\mu+\theta)(\mu+\phi)}+\mu\epsilon\rho\right)$$
$$<\mu(\mu+\delta+\gamma+\tau+\sigma)+\sigma(\delta+\gamma+\tau)$$

$$\frac{\sigma\beta\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)}\left((1-\rho)(\mu+\alpha)+\frac{\theta\alpha\rho}{(\mu+\theta)(\mu+\phi)}+\mu\epsilon\rho\right)<1.$$

That is,

$$R_o<1.$$

VII. Global Stability of the Disease-Free Equilibrium (DFE)

Theorem 2. *The DFE ξ_o is globally asymptotically stable when $R_o \leq 1$.*

Proof: We consider a suitable Lyapunov functional $V=V(S,V_1,V_2,E,I)$ defined by

$$\begin{aligned}V(S,V_1,V_2,E,I) &= \left(S-S^o-S^o\ln\frac{S}{S^o}\right)+\left(V_1-V_1^o-V_1^o\ln\frac{V_1}{V_1^o}\right)\\ &\quad+\left(V_2-V_2^o-V_2^o\ln\frac{V_2}{V_2^o}\right)+aE+bI,\end{aligned}\tag{11}$$

where

$$a=\frac{\sigma}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)}\,;b=\frac{1}{\mu+\delta+\gamma+\tau}$$

and

$$\left[S^o=\frac{1}{\mu}(1-\rho)\pi+\frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)},V_1^o=\frac{\rho\pi}{\mu+\alpha},V_2^o=\frac{\alpha\rho\pi}{(\mu+\phi)(\mu+\alpha)}\right],$$

are the values S, V_1 and V_2 at DFE.
Clearly, $a>0$ and $b>0$ in (11), for $S>S^o\exp\left(\frac{S}{S^o}\right)$, $V_1>V_1^o\exp\left(\frac{V_1}{V_1^o}\right)$ and $V_2>V_2^o\exp\left(\frac{V_2}{V_2^o}\right)$ in (11), since S^o, V_1^o, V_2^o are equilibrium points of S, V_1 and V_2. Then, the first three terms of (11) are positive.
Therefore, $V(S,V_1,V_2,E,I)$ is positive definite.
The time derivative of the function V in (11) along system (1) becomes

$$\begin{aligned}\frac{dV}{dt} &= \left(1-\frac{S^o}{S}\right)[(1-\rho)\pi-\beta SI-\mu S+\theta R]\\ &+\left(1-\frac{V_1^o}{V_1}\right)[\rho\pi-\epsilon\beta V_1 I-(\mu+\alpha)V_1]\\ &+\left(1-\frac{V_2^o}{V_2}\right)[\alpha V_1-(\mu+\phi)V_2]\\ &+a[\beta SI+\epsilon\beta V_1 I-(\mu+\sigma)E]\\ &+b[\sigma E-(\mu+\delta+\gamma+\tau)I],\end{aligned}$$

at disease-free equilibrium, we have

$$\begin{aligned}(1-\rho)\pi &= \beta SI^o+\mu S^o-\theta R^o;\\ \rho\pi &= \epsilon\beta V_1 I^o+(\mu+\alpha)V_1;\\ \alpha V_1 &= (\mu+\phi)V_2^o\\ E &= \frac{\beta SI^o+\epsilon\beta V_1 I^o}{(\mu+\sigma)};\\ \sigma E &= (\mu+\delta+\gamma+\tau)I^o.\end{aligned}$$

It follows that

$$\begin{aligned}\frac{dV}{dt} &= \left(1-\frac{S^o}{S}\right)[\beta SI^o+\mu S-\theta R^o-\beta SI-\mu S+\theta R]\\ &+\left(1-\frac{V_1^o}{V_1}\right)[\epsilon\beta V_1 I^o+(\mu+\alpha)V_1-\epsilon\beta V_1 I-(\mu+\alpha)V_1]\\ &+\left(1-\frac{V_2}{V_2}\right)[(\mu+\phi)V_2^o-(\mu+\phi)V_2]\\ &+a\left[\beta SI+\epsilon\beta V_1 I-(\mu+\sigma)\cdot\frac{(\beta SI^o+\epsilon\beta V_1 I^o)}{(\mu+\sigma)}\right]\\ &+b[(\mu+\delta+\gamma+\tau)I^o-(\mu+\delta+\gamma+\tau)I].\end{aligned}$$

Thus,

$$\begin{aligned}\frac{dV}{dt} &= \left(1-\frac{S^o}{S}\right)[\beta SI^o-\beta SI+\mu(S^o-S)-\theta(R^o-R)]\\ &+\left(1-\frac{V_1^o}{V_1}\right)[\epsilon\beta V_1 I^o-\epsilon\beta V_1 I+(\mu+\alpha)(V_1^o-V_1)]\\ &+\left(1-\frac{V_2^o}{V_2}\right)[(\mu+\phi)(V_2^o-V_2)]+a[\beta SI+\epsilon\beta V_1 I-\beta SI^o-\epsilon\beta V_1 I^o]\\ &+b(\mu+\delta+\gamma+\tau)(I^o-I).\end{aligned}$$

At DFE, $I^o=0$, therefore

$$\begin{aligned}\frac{dV}{dt} &= \left(1-\frac{S^o}{S}\right)[-\beta SI+\mu(S^o-S)-\theta(R^o-R)]\\ &+\left(1-\frac{V_1^o}{V_1}\right)[-\epsilon\beta V_1 I+(\mu+\alpha)(V_1^o-V_1)]\\ &+\left(1-\frac{V_2^o}{V_2}\right)[(\mu+\phi)(V_2^o-V_2)]\\ &+a[\beta SI+\epsilon\beta V_1 I]-b(\mu+\delta+\gamma+\tau)I\end{aligned}$$

Since $b=\frac{1}{(\mu+\delta+\gamma+\tau)}$,

It follows that

$$\begin{aligned}\frac{dV}{dt} &= \left(1-\frac{S^o}{S}\right)[-\beta SI+\mu(S^o-S)-\theta(R^o-R)]\\ &+\left(1-\frac{V_1^o}{V_1}\right)[-\epsilon\beta V_1 I+(\mu+\alpha)(V_1^o-V_1)]\\ &+\left(1-\frac{V_2^o}{V_2}\right)[(\mu+\phi)(V_2^o-V_2)]+a(\beta S+\epsilon\beta V_1)I-I.\end{aligned} \tag{12}$$

Rewriting (12), we have

$$\begin{aligned}\frac{dV}{dt} &= [a(\beta S+\epsilon\beta V_1)-1]I-\left(\frac{S-S^o}{S}\right)[\beta SI+\mu(S-S^o)-\theta(R-R^o)]\\ &-\left(\frac{V-V_1^o}{V_1}\right)[\epsilon\beta V_1 I+(\mu+\alpha)(V_1-V_1^o)]\\ &-\left(\frac{V_2-V_2^o}{V_2}\right)[(\mu+\phi)(V_2-V_2^o)],\end{aligned}$$

at DFE, $S=\frac{1}{\mu}\left[(1-\rho)\pi+\frac{\phi\alpha\rho\pi}{(\mu+\theta)(\mu+\phi)(\mu+\alpha)}\right]$; $V_1=\frac{\rho\pi}{\mu+\alpha}$

So that

$$\begin{aligned}\frac{dV}{dt} &= \left[\frac{a\beta\pi}{\mu(\mu+\alpha)}\left(\left[(1-\rho)(\mu+\alpha)+\frac{\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)}\right]+\mu\epsilon\rho\right)-1\right]I\\ &-\left(\frac{S-S^o}{S}\right)[\beta SI+\mu(S-S^o)-\theta(R-R^o)]\\ &-\left(\frac{V-V_1^o}{V_1}\right)[\epsilon\beta V_1 I+(\mu+\alpha)(V_1-V_1^o)]\\ &-\left(\frac{V_2-V_2^o}{V_2}\right)[(\mu+\phi)(V_2-V_2^o)].\end{aligned}$$

Since $a=\frac{\sigma}{(\mu+\sigma)(\mu+\delta+\gamma+\tau)}$, we have

$$\frac{dV}{dt} = \left[\frac{\beta\sigma\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)} \times \left((1-\rho)(\mu+\alpha)+\frac{\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)}+\mu\epsilon\rho\right)-1\right]I - \left(\frac{S-S^o}{S}\right)[\beta SI+\mu(S-S^o)-\theta(R-R^o)] - \left(\frac{V-V_1^o}{V_1}\right)[\epsilon\beta V_1 I+(\mu+\alpha)(V_1-V_1^o)] - \left(\frac{V_2-V_2^o}{V_2}\right)[(\mu+\phi)(V_2-V_2^o)]$$

That is,

$$\frac{dV}{dt} = (R_o-1)I - \left(\frac{S-S^o}{S}\right)[\beta SI+\mu(S-S^o)-\theta(R-R^o)] - \left(\frac{V-V_1^o}{V_1}\right)[\epsilon\beta V_1 I+(\mu+\alpha)(V_1-V_1^o)] - \left(\frac{V_2-V_2^o}{V_2}\right)[(\mu+\phi)(V_2-V_2^o)],$$

where

$$R_o = \frac{\beta\sigma\pi}{\mu(\mu+\sigma)(\mu+\delta+\gamma+\tau)(\mu+\alpha)}\left((1-\rho)(\mu+\alpha)+\frac{\phi\alpha\rho}{(\mu+\theta)(\mu+\phi)}+\mu\epsilon\rho\right).$$

$\frac{dV}{dt}=0$ if and only if $S=S^o, V_1=V_1^o, V_2=V_2^o, I=0$ and $\frac{dV}{dt}<0$ if $R_o<1$. The only compact invariant set is the single-element ξ_o. Thus, according to Lasalle's invariant principle, for any initial condition in the model system (1), as $t\to\infty$, the solution will approach ξ_o if $R_o\le 1$. Therefore, when $R_o\le 1$, the disease-free equilibrium is globally asymptotically stable.

VIII. Numerical Simulations and Results

The model was assessed through numerical analysis, and simulations allowed for the observation of parameter impacts. Wolfram Mathematica was utilized for conducting the simulations.

The table below contains the parameter values used to study how a double dose of vaccine and an effective contact rate impact the speed at which diarrhea spreads in a population of susceptible, vaccinated 1, vaccinated 2, exposed, infected, and recovered individuals. The value of the rate of acquiring immunity from the second dose of vaccine was varied first, followed by the treatment rate (τ). Table 2. Values of the Parameters for Figures 3 - 6.

Table 2. Parameter values

Parameters	Values	Source
π	1000	Assumed
β	0.000009	[X]
μ	0.012	[I]
θ	0.2	Assumed
ϵ	0.1	Assumed
α	0.3	Assumed
ϕ	0.2	Assumed
τ	0.1	[I]
σ	0.7	[I]
δ	0.5	Assumed
γ	0.6	Assumed

IX. Sensitivity Analysis

The table below shows the sensitivity index of each parameter when their values are inputted into the partial differential equations and solved,

$$X_p^{R_o} = \frac{\partial}{\partial p}(R_o) \times \frac{p}{R_o}$$

and X^{R_o} being the sensitivity of R_o concerning any parameter. The sensitivity index for the parameters, as determined by the derivative-based local method, is depicted in Figure 2.

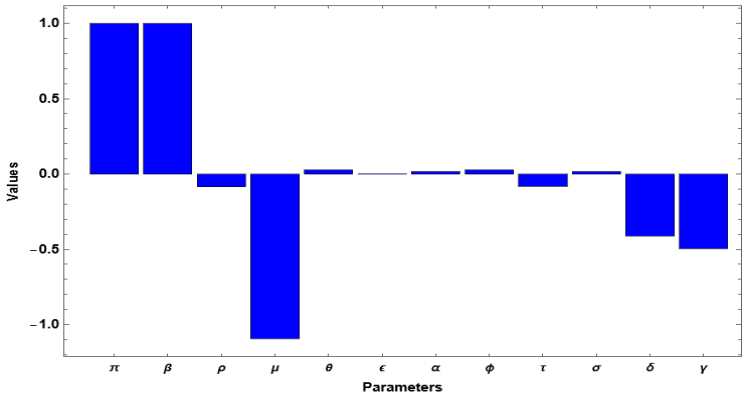
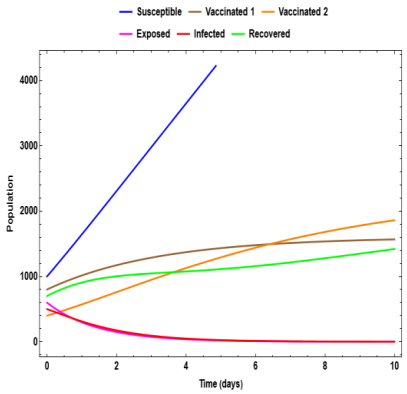
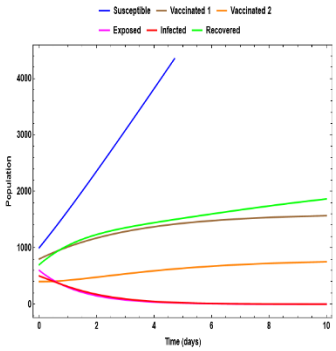


Fig. 2. Sensitivity index of parameters of R_o.

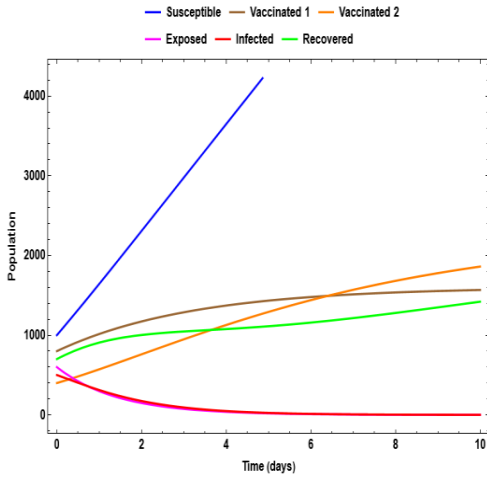


(a) Graph of population against time for $\pi = 1000, \beta = 0.000009, \rho = 0.5, \mu = 0.012, \theta = 0.2, \varepsilon = 0.1, \alpha = 0.3, \phi = 0.2, \gamma = 0.6, \tau = 0.1, \sigma = 0.7, \delta = 0.5$

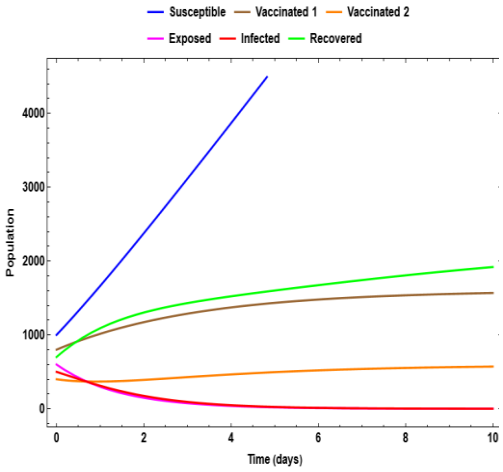


(b) Graph of population against time for $\pi = 1000, \beta = 0.000009, \rho = 0.5, \mu = 0.012, \theta = 0.2, \varepsilon = 0.1, \alpha = 0.3, \phi = 0.6, \gamma = 0.6, \tau = 0.1, \sigma = 0.7, \delta = 0.5$

Fig. 3. Effect of acquiring immunity from second dose vaccine

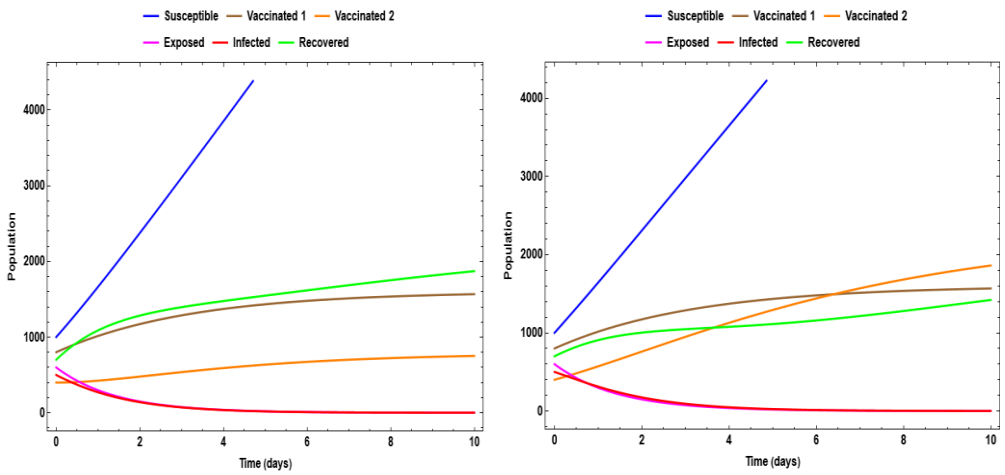


(a) Graph of population against time for $\pi = 1000, \beta = 0.000009, \rho = 0.5, \mu = 0.012, \theta = 0.2, \varepsilon = 0.1, \alpha = 0.3, \phi = 0.6, \gamma = 0.7, \tau = 0.1, \sigma = 0.7, \delta = 0.5$



(b) Graph of population against time for $\pi = 1000, \beta = 0.000009, \rho = 0.5, \mu = 0.012, \theta = 0.2, \varepsilon = 0.1, \alpha = 0.3, \phi = 0.8, \gamma = 0.6, \tau = 0.1, \sigma = 0.7, \delta = 0.5$

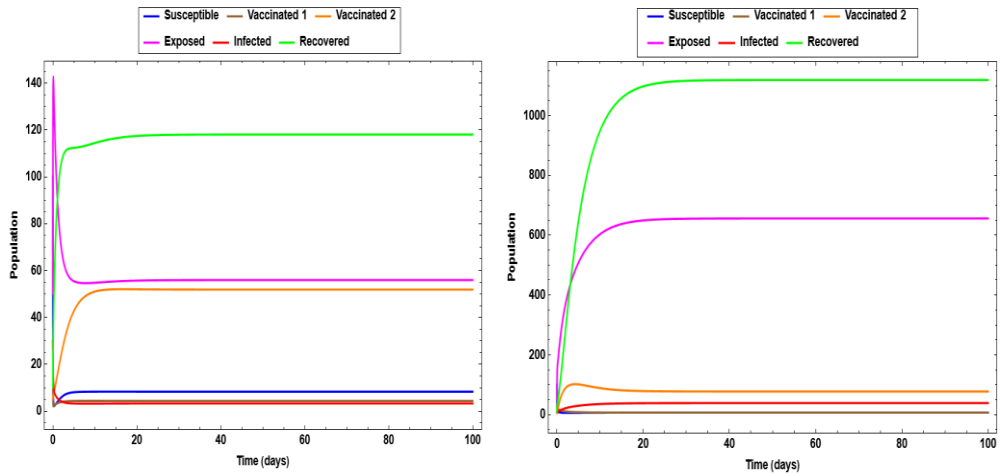
Fig. 4. Effect of acquiring immunity from second dose vaccine



(a) Graph of population against time for $\pi = 1000, \beta = 0.000009, \rho = 0.5, \mu = 0.012, \theta = 0.2, \varepsilon = 0.1, \alpha = 0.3, \phi = 0.2, \gamma = 0.6, \tau = 0.1, \sigma = 0.7, \delta = 0.5$

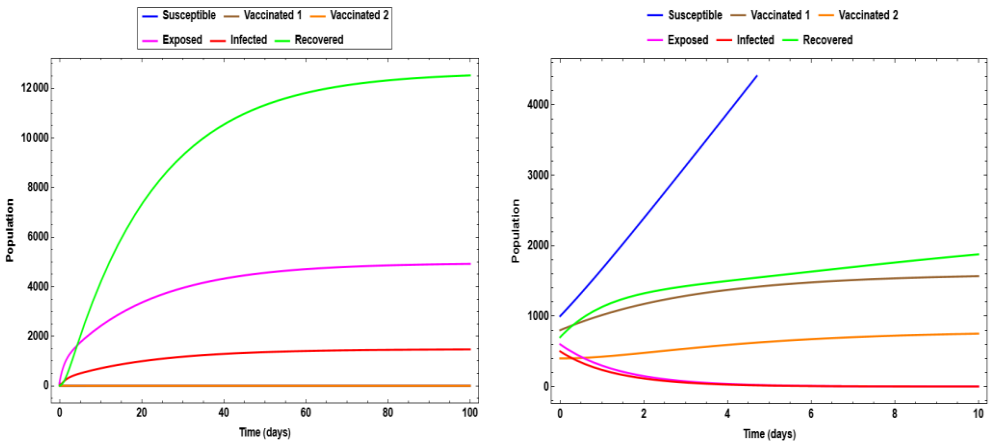
(b) Graph of population against time for $\pi = 1000, \beta = 0.000009, \rho = 0.5, \mu = 0.012, \theta = 0.2, \varepsilon = 0.1, \alpha = 0.3, \phi = 0.2, \gamma = 0.6, \tau = 0.3, \sigma = 0.7, \delta = 0.5$

Fig. 5. Effect of treatment rate on the dynamics of diarrhea disease



(a) π *(b)* π

Fig. 6. The dependence of R_o on π, β, ϕ



(a) β *(b)* ϕ

Fig. 7. The dependence of R_o on π, β, ϕ

X. Discussion

In this work, we studied the impact of a double-dose vaccine on the dynamics of diarrhea disease using a mathematical model. The disease-free and endemic equilibria and the basic reproduction number R_0 were all determined. The result of the qualitative analyses showed that disease-free equilibrium is locally asymptotically stable if $R_o < 1$ and unstable otherwise. The implication of this is: If the basic reproduction number is less than one, diarrhea can be managed by administering a double dose vaccine, regardless of the initial number of infected individuals in the population. However, if the reproduction number exceeds unity, then diarrhea will persist in the population. We also obtained conditions under which the disease-free equilibrium is globally stable. If the equilibrium point is globally stable, it means that when a population is initially free of the disease, it will remain disease-free in the long run. This suggests that effective control measures have been implemented to prevent the reintroduction or spread of the disease.

The sensitivity analysis in Figure 2 showed that the contact rate. (β) and the recruitment rate (π) are the most sensitive parameters of the basic reproduction number R_o, with a positive index. This means that changes in the values of β and π have the greatest effect on the reproduction number, and consequently, on the prevalence of the disease in the population. The result $\chi_{\beta}^{R_o} = 1.0$ and $\chi_{\pi}^{R_o} = 1.0$ implies if β and π is increased (decreased) by 10 percent then R_o will also increase(decrease) by 10 percent. Also very sensitive is the infectivity rate of the exposed individuals (σ) and rate of immunity from the second dose (ϕ) . The result $\chi_{\sigma}^{R_o} = 0.0168539$ and $\chi_{\phi}^{R_o} = 0.0260482$ implies if σ and ρ are increased (decreased) by 10 percent the R_o increase (decrease) by 0.169 percent and 0.261 percent respectively. The dependence of R_o on parameters π, β, and ϕ are clearly indicated in Figure 6. (a) and (b) and Figure 7. (a) and (b) respectively where the infected population reduced drastically with a significant increase in the number of recovered individuals. The sensitivity indices of the other parameters can be interpreted similarly.

The graphs of the simulation are shown in Figures 3 - 4. In Figure 3. (a) and (b) and Figure 4. (a) and (b), the effect of acquiring immunity from second dose vaccine (ϕ) were shown. The findings indicate that an increase in the rate of immunity acquisition from the second dose of vaccine leads to an increase in the susceptible and recovered classes and a decrease in infections. This suggests that as individuals acquire immunity from the second dose of vaccine, the occurrence of infections decreases. Therefore, by increasing immunity through the second dose, it is possible to reduce the number of individuals who are exposed to and infected with diarrhea. in addition, the impact of the treatment rate (τ) on the dynamics of diarrhea, the disease was investigated and the results are displayed in Figure 5. (a) and (b) which showed the higher the treatment rate the lower the infected population. This means that as the treatment rate goes up, the number of people who are unable to recover in the population goes down. Moreover, an increase in the treatment rate results in a corresponding rise in the number of individuals who have recovered.

X. Conclusion

To effectively combat diarrhea disease in the population, health authorities, health caregivers, and the community must work together to reduce the basic reproduction number (R_o) below one. This can be done by decreasing contact rates, limiting travel or visits to areas where diarrhea disease is prevalent, and ensuring that vaccines and vaccination programs are effective and widely available as shown in the sensitivity analysis conducted in this study. Additionally, individuals should be encouraged to receive a double dose of the vaccine to prevent future infections.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

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